New continuous approximation models for passenger and freight transportation

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Project Objective

The purpose of this project has been to design simple and concise mathematical models for quantifying the reductions in vehicle miles travelled (VMT), among other cost measures, that result from implementing modern logistics systems with complex features such as ridesharing, crowdsourcing or teleworking. Traditionally, these problems have been solved in a discrete setting, involving fixed sets of (for example) demand points, time periods, and service facility locations; one then solves them with an integer mathematical programming solver. A drawback of this approach is that solving large-scale instances would require enormous computational efforts which likely increase exponentially with the problem instance size. A further drawback is that such models are often extremely complex, which hinders understanding of salient problem features and managerial insights.

For these reasons, this project has used tools from geospatial optimization, computational geometry, and geometric probability theory to discover simple *continuous approximation models* that identify the key problem attributes that affect them most significantly. A continuous approximation model is characterized by its use of continuous representations of input data and decision variables as density functions over time and space, and the goal is to approximate the objective function into an expression that can be optimized by relatively simple analytical operations. The results from such models often bear closed-form analytical structures that help reveal managerial insights.

Problem Statement

One family of problems that we found particularly relevant for this study are what we call *selection routing problems*. A selection routing problem is a routing optimization problem, such as the travelling salesman problem (TSP), in which one is given a large collection of destinations and the goal is to select a subset of those points that satisfies certain criteria and optimizes some objective function. Such problems are particularly timely in modern analysis of logistical systems in several contexts:

- Selection routing problems arise organically in studying the consequences of trip chaining [24], that is, performing multiple errands during a single outing, because one has multiple choices of locations at which to perform errands.
- One proposed approach for mitigating the inefficiencies in "last mile" delivery has been the use of a socially networked system in which parcel recipients can "opt in" for packages to be delivered at multiple possible locations (as opposed to their doorstep), such as their workplace.
- Selection routing problems are fundamentally important in studying *randomized strategies* in warehouses, in which one stores a stock keeping unit (SKU) in any available location (as opposed to designating specific regions of the warehouse for different SKUs). This is because a warehouse picker will often select multiple SKUs at a time and can benefit if those SKUs are dispersed throughout the warehouse.

Research Methodology

We developed continuous approximation formulas for two selection routing problems that arise in modern logistical systems, the *one-of-a-subset travelling salesman problem* (TSP) and the *cardinality-constrained TSP*, which have applications in last mile delivery and ridesharing. Our approach was to prove mathematical theorems about the costs of these problems in an asymptotic limit as demand becomes large, then verify that those theorems predict the costs correctly in real-world experiments.

Results

We proved two theorems that allowed us to predict the tour lengths (in terms of VMT) of the one-of-a-subset TSP and the cardinality-constrained TSP, both of which are difficult to solve explicitly. We then compared our predictions with real-world simulations, both in the Euclidean plane and in a road network, and found them to be of high quality. Our main conclusion is that for both problems, there exist simple closed-form solutions that accurately estimate their cost that can be easily trained, for both Euclidean instances and instances in a road network.

Figure 1: Random stow in a warehouse, with different items located amongst one another.

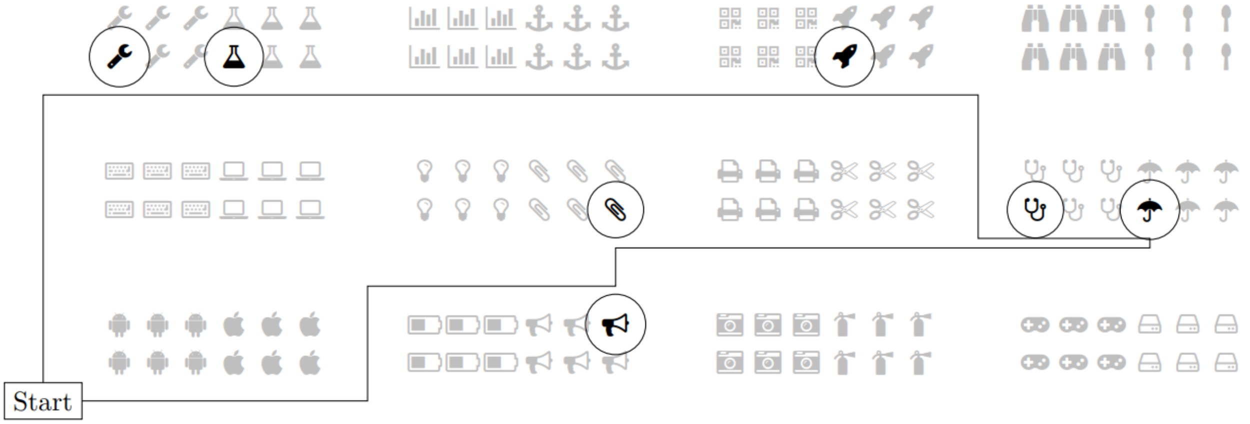

Figure 2 A warehouse layout using non-random stow, with SKUs stored together in groups. The path shown is the shortest one that collects the 7 circled items.

Figure 3 A warehouse layout using random stow, with SKUs stored randomly. The path shown is the shortest one that collects the 7 circled items; notice it is significantly shorter than that in the previous figure.